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**TECHNICAL NOTE 2087**

COMPARISON OF THEORETICAL AND EXPERIMENTAL HEAT  
TRANSFER ON A COOLED  $20^\circ$  CONE WITH A LAMINAR  
BOUNDARY LAYER AT A MACH NUMBER OF 2.02

By Richard Scherrer and Forrest E. Gowen

Ames Aeronautical Laboratory  
Moffett Field, Calif.



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SUMMARY

Heat-transfer measurements have been made on an air-cooled  $20^\circ$  cone with a laminar boundary layer at a Mach number of 2.02. The surface temperature in the instrumented area on the cone was essentially constant at all test conditions; however, this area was preceded by severe surface-temperature gradients. Although these gradients affected the heat transfer in the instrumented area, the effect decreased with length. A comparison of the theory for a constant surface temperature with the experimental results on the basis of the heat-transfer parameter defined as the Nusselt number divided by the square root of Reynolds number indicates agreement within the experimental accuracy over 60 percent of the instrumented length on the cone.

INTRODUCTION

The problem of calculating the cooling requirements for supersonic aircraft has recently become important, and numerous theoretical and experimental investigations of heat transfer at supersonic speeds have been conducted. The major theoretical works prior to 1947 have been conveniently summarized by Johnson and Rubesin (reference 1) and comparisons between these theories and experimental data have been made in references 2 and 3. Since the results presented in reference 2 indicated that surface-temperature gradients affect local rates of heat transfer, the theoretical analyses of Crocco and Hantzsche and Wendt have been extended by Chapman and Rubesin to include the effect of an arbitrary distribution of surface temperature. (See reference 4.)

In the investigation of heat transfer on bodies with a laminar boundary layer, almost all the experimental studies have been performed with heated models because of the resulting simplicity of construction and instrumentation. Although these investigations are satisfactory

for a partial comparison between theory and experiment, tests with a cooled model are necessary for a complete comparison at the conditions which usually occur in supersonic flight.

The purpose of the investigation described in this report was to obtain heat-transfer data with a laminar boundary layer and with surface cooling for comparison with the theories of references 1 and 4.

### SYMBOLS

The following symbols have been used in the presentation of the theoretical and experimental data:

- A    area, square feet
- $C_d$    skin-friction drag coefficient
- $c_p$    specific heat at constant pressure, Btu per pound, degree Fahrenheit
- $g$     gravitational constant (32.2), feet per second squared
- $H$     total pressure, pounds per square inch absolute
- $h$     local heat-transfer coefficient  $\left[ \frac{Q}{A(T_R - T_s)} \right]$ , Btu per hour, square foot, degree Fahrenheit
- $k$     thermal conductivity, Btu per hour, square foot, degree Fahrenheit per foot
- $l$     body length, feet
- $M$     Mach number, dimensionless
- $Nu$    local Nusselt number  $\left( \frac{hs}{k_v} \right)$ , dimensionless
- $p$     static pressure, pounds per square foot absolute
- $Pr$    Prandtl number  $\left( \frac{3600g c_p \mu_v}{k_v} \right)$ , dimensionless
- $Q$     rate of heat transfer, Btu per hour
- $Re$    local Reynolds number  $\left( \frac{\rho_v V_s}{\mu_v} \right)$ , dimensionless

- $Re_l$  Reynolds number based on body length
- $s$  distance from nose along body surface, feet
- $T$  temperature, degrees Fahrenheit
- $\Delta T$  temperature increment, degrees Fahrenheit
- $T_R$  recovery temperature (surface temperature at condition of zero heat transfer), degrees Fahrenheit
- $V$  velocity just outside the boundary layer, feet per second
- $W$  flow rate of coolant, pounds per minute
- $x$  distance from the nose along the body axis, feet
- $x^*$  dimensionless length  $\left(\frac{x}{l}\right)$
- $\mu$  absolute viscosity, pound-seconds per square foot
- $\rho$  air density, slugs per cubic foot

### Subscripts

In addition, the following subscripts have been used:

- $s$  conditions at the body surface
- $v$  fluid conditions just outside the boundary layer
- $o$  stagnation conditions in the free stream
- $c$  coolant

### APPARATUS

The experimental investigation was made with an air-cooled cone in the Ames 1- by 3-foot supersonic wind tunnel No. 1. This closed-circuit, variable-density wind tunnel is equipped with a nozzle having flexible top and bottom plates which can be shaped to give test-section Mach numbers in the range of 1.2 to 2.4. The total-pressure level in the wind tunnel can be varied from 1/5 of an atmosphere to 3 atmospheres

absolute depending on the Mach number and the ambient temperature. The air in the wind tunnel is dried to an absolute humidity of 0.0001 pound of water per pound of dry air in order to make negligible the effects of condensation in the nozzle. The cone, which is shown in figure 1, was installed on a sting in the center of the test section and the coolant pipes were brought to the cone from the rear through a thin cantilevered strut mounted on the side wall of the wind tunnel.

### Cooling System

The primary requirement of the cooling system was that it provide an adjustable and stable outlet temperature at a constant coolant flow rate. This requirement was satisfied by the cooling system shown schematically in figure 2. The clean, dry air which was used as the coolant in the cone was obtained from the make-up air system of the wind tunnel at a pressure of about 90 pounds per square inch absolute. This air first passed through a pressure regulator, which was used to maintain a constant air-flow rate, and was precooled in the recovery heat exchanger. It was then cooled to a minimum temperature of  $-90^{\circ}$  F in the air-to-alcohol heat exchanger, passed into the cone and back through the recovery heat exchanger, and finally was discharged to the atmosphere through a rotameter which was used to indicate the flow rate of the coolant.

The cooling was produced by a mixture of dry ice and alcohol in sufficient quantity to maintain a constant coolant temperature at the maximum rate of heat transfer to the cone. The alcohol, which was circulated in the air-to-alcohol heat exchanger, was cooled in a coil of copper tubing submerged in the constant-temperature bath. The alcohol was circulated in the cooling system by an aircraft hydraulic pump driven by an electric motor. Control of the air temperature in the system was obtained by controlling the alcohol flow rate with the valves shown in figure 2.

A considerable reduction in the rate of dry-ice consumption and an increase in the thermal stability of the system was achieved by the use of the recovery heat exchanger. This was due to the fact that the temperature rise of the coolant in the cone and in the inlet and outlet pipes was less than half the difference between the minimum coolant temperature and the temperature of the air supply.

### Air-Cooled Cone

The construction details of the cone are shown in figure 1. The outer shell was made of stainless steel and had a wall thickness of only 0.028 inch in order to minimize the heat conduction along the shell in the instrumented area. The surface of the outer cone was highly polished

in order to reduce radiant heat transfer and to prevent local transition in the boundary layer due to roughness. The inner cone also was made of stainless steel and had a wall thickness of 0.070 inch which was thin enough to make all conduction effects in the inner cone negligible.

Cooling air entered the cone through a tube connected to the hollow sting and flowed forward through the center tube to the cone tip. From the chamber at the tip, the air flowed aft through the narrow annular passage between the inner and outer cones into the rear chamber and out the exhaust line to the recovery heat exchanger in the cooling system. The annular gap between the cones was designed to give a uniform surface temperature in the test region ( $x^* = 0.4$  to  $0.8$ ) so that the experimental results could readily be compared with theory. Turbulent flow in this annular gap was necessary to obtain the most accurate readings of local coolant temperature; therefore, the heat-transfer system was designed to operate at a Reynolds number sufficiently high to insure turbulent flow. In addition, the outer surface of the inner cone was roughened with 1/32-inch-wide circumferential grooves spaced at 1/4-inch intervals. During the investigation, tests were made with coolant flow rates of 70 percent and 140 percent of the design value to determine the effect of changes in the internal-flow Reynolds number on the internal-temperature measurements. The results indicated that this effect was negligible.

Another body, with external dimensions identical to those of the heat-transfer cone, was equipped with pressure orifices. This cone was used to obtain measurements of the local pressures which existed on the surface of the heat-transfer cone during the tests in order to have a measure of the quality of the air stream.

### Instrumentation

The method by which local rates of heat transfer were obtained consisted of the measurement of the temperature change of a known mass flow of coolant along an increment of the body length. In order to measure this coolant temperature change accurately with thermocouples, it was necessary to have a large temperature rise at a given rate of heat transfer. Therefore, a fluid of low specific heat was required. Air was selected because of its low specific heat and because it was available at the desired pressure. The coolant temperature was measured at five stations spaced approximately 1 inch apart along the gap between the inner and outer cones by iron-constantan thermocouples. Three thermocouple junctions were located 120° apart midway between the walls of the coolant passage at each of the five longitudinal stations. These junctions were installed in micarta plugs, threaded into the shell of the inner cone, in order to reduce the error in the temperature measurements due to conduction. The thermocouple lead wires were sealed in the surface of the inner cone and at the base of the cone in order to prevent air leakage. The thermocouple potentials were measured with a

sensitive potentiometer, and an external null-reading galvanometer was used to indicate the balance of the circuits.

The surface temperatures were measured by stainless-steel-constantan thermocouples located on two rays of the cone 180° apart. Twelve thermocouples were placed 0.5 inch apart in rows arranged so that the longitudinal temperature distribution, using data from both sides, could be measured at 0.25-inch-length increments. This arrangement provided some information on the circumferential temperature distribution as well as the longitudinal distribution. The thermocouples consisted of fine constantan wires which were soldered into the outer shell of the cone and a common stainless-steel lead wire which was attached to the base of the outer shell by means of a stainless-steel screw. The constantan leads between the thermocouple junctions and the base of the cone were flattened to a thickness of 0.002 inch and cemented to the inner surface of the cone with a thin coat of insulating cement. This installation minimized the interference of the leads with the flow of air in the coolant passage. At the base of the cone, the flattened wires were arranged so that a sliding contact was made with the main constantan lead wires. (See fig. 1.) These lead wires and the common stainless-steel wire were connected through an automatic switch to a potentiometer which automatically recorded the surface-temperature distribution.

The coolant flow rate was measured with the rotameter at the point where the coolant was discharged to the atmosphere.

#### TEST PROCEDURE

The first phase of the investigation consisted of measuring the local pressures on the surface of the pressure-distribution cone. Then the heat-transfer cone was installed in exactly the same position in the wind-tunnel test section and the heat-transfer phase of the investigation was undertaken. The tests were performed over a range of surface temperatures in order to define the relationship between Nusselt number and Reynolds number for the cooled cone. The investigation was performed only at a nominal Mach number of 2.0 because theory indicates that the effect of Mach number on heat-transfer parameter is small within the available range (1.2 to 2.4), and the most uniform flow in the wind tunnel is obtained at this Mach number.

The test conditions for the investigation are shown in the following table:

Heat-transfer test conditions (Mach number, 2.02)			
$Re_L \times 10^{-6}$	$(T_s + 460)/(T_R + 460)$		
2.22	0.898	0.868	0.821
3.64	.905	.868	.814
5.01	.914	.874	.834

Schlieren observations, liquid-film tests, and the absence of discontinuities in the temperature-distribution measurements that would denote transition indicated that the boundary layer remained laminar for all the test conditions. At any set of test conditions, the wind-tunnel temperature and pressure and the cone temperature were allowed to reach equilibrium after which the following data were recorded:

Total temperature

Total pressure

Test-section static pressure

Absolute humidity

Cone surface temperatures

Coolant temperatures in the cone

Coolant flow rate

Data were taken at three nominal surface temperatures at each pressure, one at the minimum temperature obtainable with the cooling system and at approximately 20° and 40° F increments above the minimum temperature. The recovery temperature was obtained by shutting off the internal air flow and allowing the surface temperature to reach equilibrium.

Because of the large mass of the wind tunnel and its components, considerable time was required to attain steady-state conditions. For this reason, one of the most important steps in the test procedure was the determination of the time at which the test conditions were stable. This was done by continuously observing the cone surface temperature and the total temperature of the wind-tunnel air stream. When these temperatures became almost constant, complete sets of data were taken continuously until at least two successive identical sets were obtained. This procedure resulted in test data being obtained for steady-state conditions.

#### ACCURACY OF RESULTS

The accuracy of the final results is based on the accuracy of the individual measurements involved and on the probable uncertainty of some of the measurements due to peculiarities of the test apparatus. The



known accuracy of the individual measurements in this investigation is as follows:

Total temperature, $T_o$ . . . . .	$\pm 1.5^\circ \text{ F}$
Surface temperature, $T_s$	
At maximum $T_s$ . . . . .	$\pm 1.5^\circ \text{ F}$
At minimum $T_s$ . . . . .	$\pm 4.5^\circ \text{ F}$
Coolant-temperature increment, $\Delta T_c$	
At maximum $T_s$ . . . . .	$\pm 0.07^\circ \text{ F}$
At minimum $T_s$ . . . . .	$\pm 0.05^\circ \text{ F}$
Internal air-flow rate, $W$ (at design flow rate). . . . .	$\pm 1.4\%$
Cone dimensions . . . . .	$\pm 0.002 \text{ in.}$
Cone-segment surface areas . . . . .	$\pm 1.4\%$
Total pressure, $H_o$ . . . . .	$\pm 0.01 \text{ psi}$
Static pressure, $p$ . . . . .	$\pm 0.01 \text{ psi}$

Although the surface-temperature thermocouples were calibrated to an accuracy of  $\pm 0.5^\circ \text{ F}$ , the experimental scatter along the curve was found to exceed this value probably because of a nonuniform coolant distribution. The local values of surface temperature used in the reduction of the test data were obtained from curves faired through the experimental data. The data point farthest from the faired curve was used in each case to indicate the probable maximum uncertainty of the surface-temperature measurement for each set of test conditions. The uncertainties in the surface temperature and internal temperature measurements accounted for approximately two-thirds of the total uncertainty in the results.

The effect of radiant heat transfer has been investigated with a similar model over a similar range of temperature differences between the cone and the wind-tunnel walls (reference 2), and was found to be negligible. The effects of longitudinal and circumferential heat conduction in the thin stainless-steel outer shell and the effects of conduction in the stainless-steel inner cone have been neglected aft of the 50-percent-length station. An approximate calculation of the actual conduction along the inner cone and through the inner cone shell revealed that these effects were only about three-tenths of 1 percent of the total heat transfer. Other sources of error such as

the small pressure gradient ( $\pm 1$  percent of the average dynamic pressure) in the test section of the wind tunnel have been neglected in the reduction of the test data.

The over-all accuracy of the final values of heat-transfer parameter was calculated from the accuracy or uncertainty of each of the individual measurements for all test conditions by the method of reference 5. Because of the variation in the uncertainty of the surface temperature and internal-temperature increments, the accuracy of the results increased somewhat along the cone length. The over-all accuracy is estimated to be  $\pm 10$  percent.

## RESULTS AND DISCUSSION

All the surface-temperature distributions obtained during the test were substantially similar and are shown in figure 3. The surface-temperature distributions are believed to start at the recovery temperature at the cone tip because the tip was solid and essentially uncooled. The rapid reduction of surface temperature ahead of the 25-percent-length station was due to the beginning of internal convective cooling in this region. The variation in the surface temperature between the 25- and 40-percent-length station was due to the sharp-edged entry to the internal annular passage that resulted in a local high-velocity region at the entrance. This effect increased the heat transfer in this region and caused the surface temperature to fall below that on the rear portion of the cone. The small surface-temperature variation between the top and bottom of the cone probably resulted from an uneven distribution of flow in the annular gap between the inner and outer cones. This variation caused some loss in accuracy in the calculation of the final results and has been included in the estimated uncertainty of the experimental data.

Results of the investigation were calculated in the form of the heat-transfer parameter,  $Nu/\sqrt{Re}$ , for the test conditions shown in the table, page 6. For each of these test conditions, measurements of flow rate of coolant and incremental coolant temperature rise were used to calculate the incremental rates of heat transfer  $Q$ . Heat-transfer coefficients were obtained by dividing the rates of heat transfer by the temperature potential ( $T_R - T_S$ ) and the surface area of the respective 1-inch segment. The heat-transfer coefficients thus obtained were considered to be local values. Local heat-transfer parameter values were then calculated from the heat-transfer coefficients, the distances along the surface from the tip of the cone to the center of the respective segment, and the fluid properties just outside the boundary layer. The results calculated in this manner are presented in figure 4. A correction for the temperature gradient that existed over part of the segment nearest the tip resulted in a conduction loss of  $3\frac{1}{2}$  to 4 percent from this segment due to conduction of heat in the outer shell of the cone toward the tip. Downstream of the

first element there was no longitudinal conduction correction because the surface temperature downstream of this point was constant.

The calculations for the constant-surface-temperature theory were based upon the following formula for a flat plate which is given in reference 1:

$$\frac{Nu}{\sqrt{Re}} = \frac{C_d \sqrt{Re}}{2} Pr^{1/3} \quad (1)$$

Hantzsche and Wendt showed that this formula may be applied to cones when modified by the factor  $\sqrt{3}$ . For the test conditions of this investigation, the value of  $C_d \sqrt{Re}$  given in reference 1 is 0.635. Thus, if Prandtl number is assumed to be 0.72, the above equation becomes

$$\frac{Nu}{\sqrt{Re}} = 0.49 \quad (2)$$

for a cone at the given test conditions. The scatter in the experimental data is about equal to the estimated maximum percent error which resulted from the inaccuracy of the measurements of the incremental changes in coolant temperature along the heat-transfer passage and from the scatter of the surface-temperature data. The experimental curve shown in figure 4 represents the numerical average of all the local heat-transfer parameter data. The theoretical curve was calculated from equation (2). The data nearest the nose deviate quite markedly from theory, a fact which is attributed to the effects of surface-temperature gradients on the laminar boundary layer. The data in this region have been corrected for the effect of conduction in the cone shell and the resulting curve is shown as a dotted line in figure 4. The values of heat-transfer parameter at each pressure show a slight increase with increasing surface temperature. This trend of the data is possibly accidental, for it is well within the experimental accuracy.

Numerous attempts were made to calculate the local values of heat-transfer parameter by the method developed in reference 4 so as to evaluate the effect of the surface-temperature gradients near the tip of the cone. The method requires that the surface-temperature distribution on the cone be transformed to the equivalent distribution for a flat plate by the method of reference 6 and then be expressed as a power-series polynomial. After an extensive investigation of methods of obtaining the desired polynomial, it was concluded that the theory of reference 4 could not be applied to the present data because the required polynomial would contain an extremely large number of terms, and for all practical purposes no useful equation could be obtained.

The surface-temperature distribution on the cone, when transformed to that of a flat plate, clearly indicates that the area in which the gradients exist is but a small part of the total area. For this reason,

their effect would be expected to decrease rapidly in the direction of the air flow. Toward the base, the experimental data would be expected to approach the value of heat-transfer parameter given by the theory for uniform surface temperature. The average curve through all the data in figure 4 illustrates this trend very clearly and, within the estimated experimental accuracy, theory and experiment are in agreement over 60 percent of the instrumented length of the cone.

#### CONCLUDING REMARKS

Tests of a cooled  $20^\circ$  cone with a laminar boundary layer show that the theory based on the assumption of a uniform surface temperature will accurately predict values of heat-transfer parameter  $Nu/\sqrt{Re}$  over the part of the cone where the effects of surface-temperature gradient are negligible. The temperature gradients in the nose region on the cone are believed to have caused the decrease in the values of the heat-transfer parameter in the forward part of the constant-temperature test region, but their effect decreased with length. Evaluation of temperature gradient effects by the method of Chapman and Rubesin (reference 4) was found to be impractical because of the great number of terms in the required analytical expression for the surface-temperature distribution.

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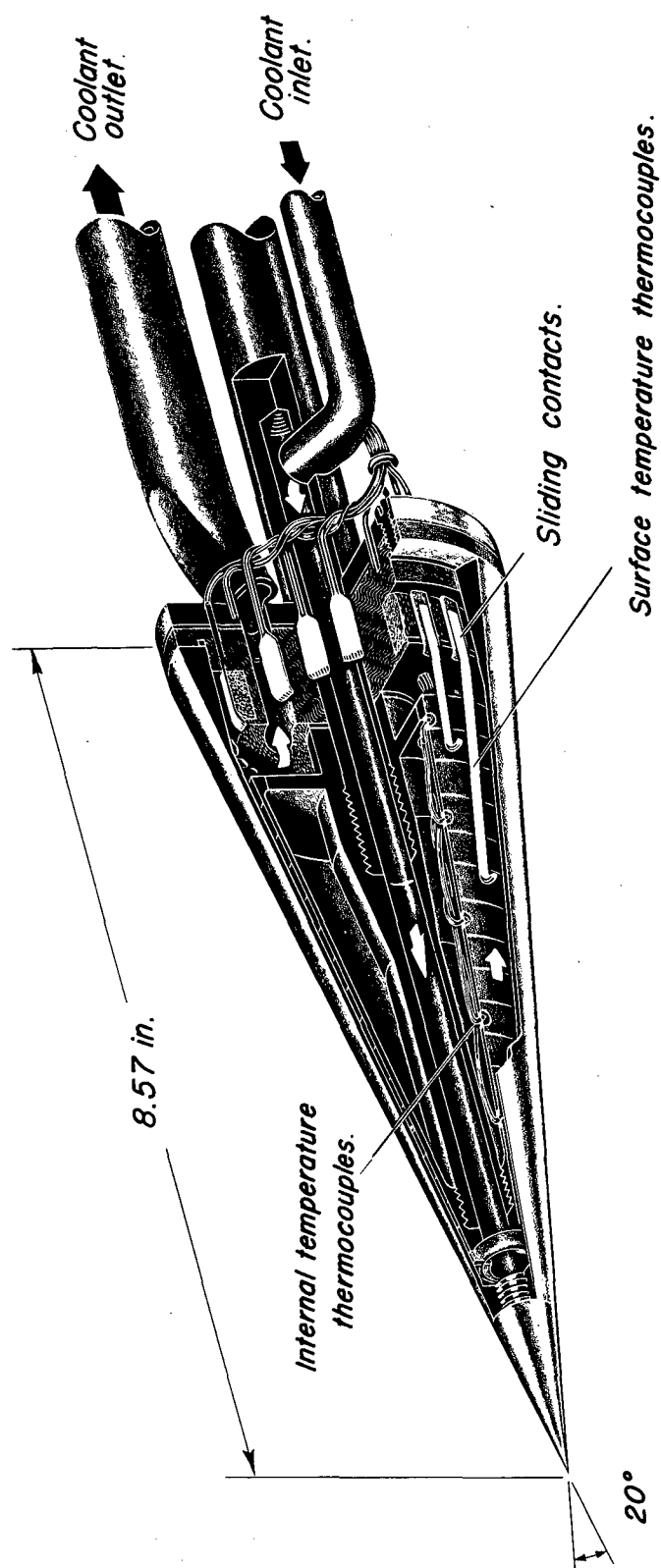


Figure 1.—Air-cooled cone.

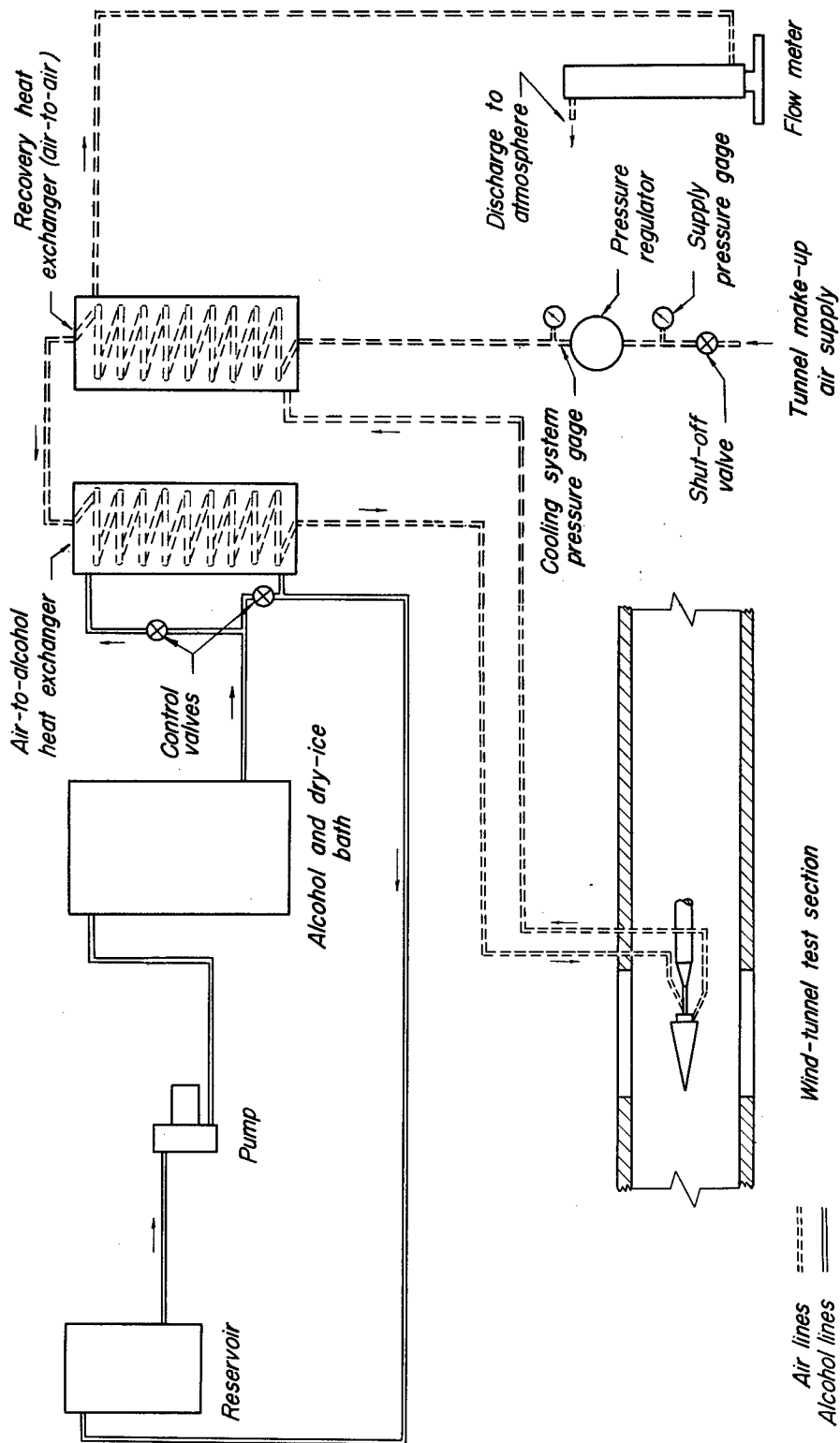
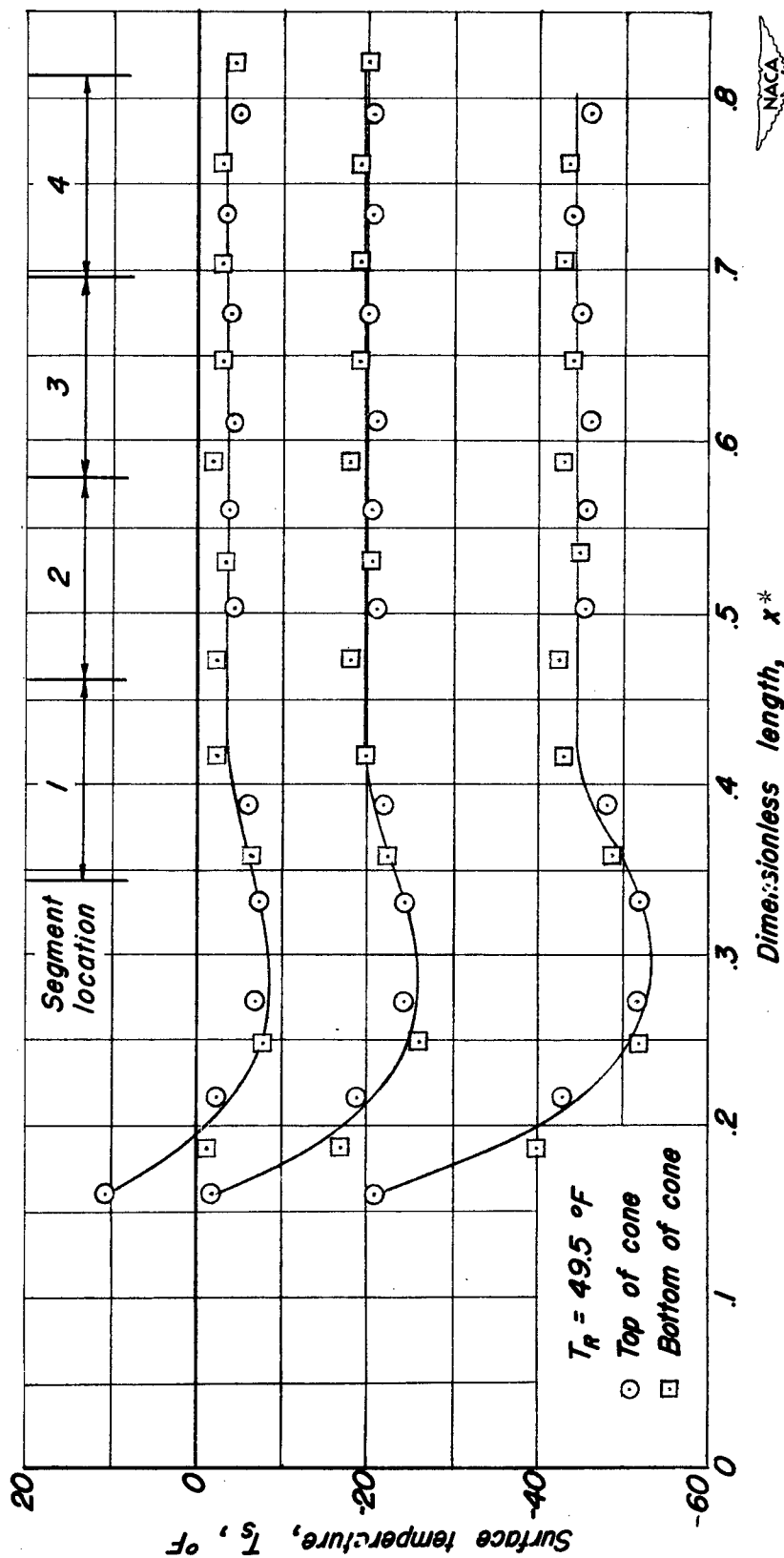


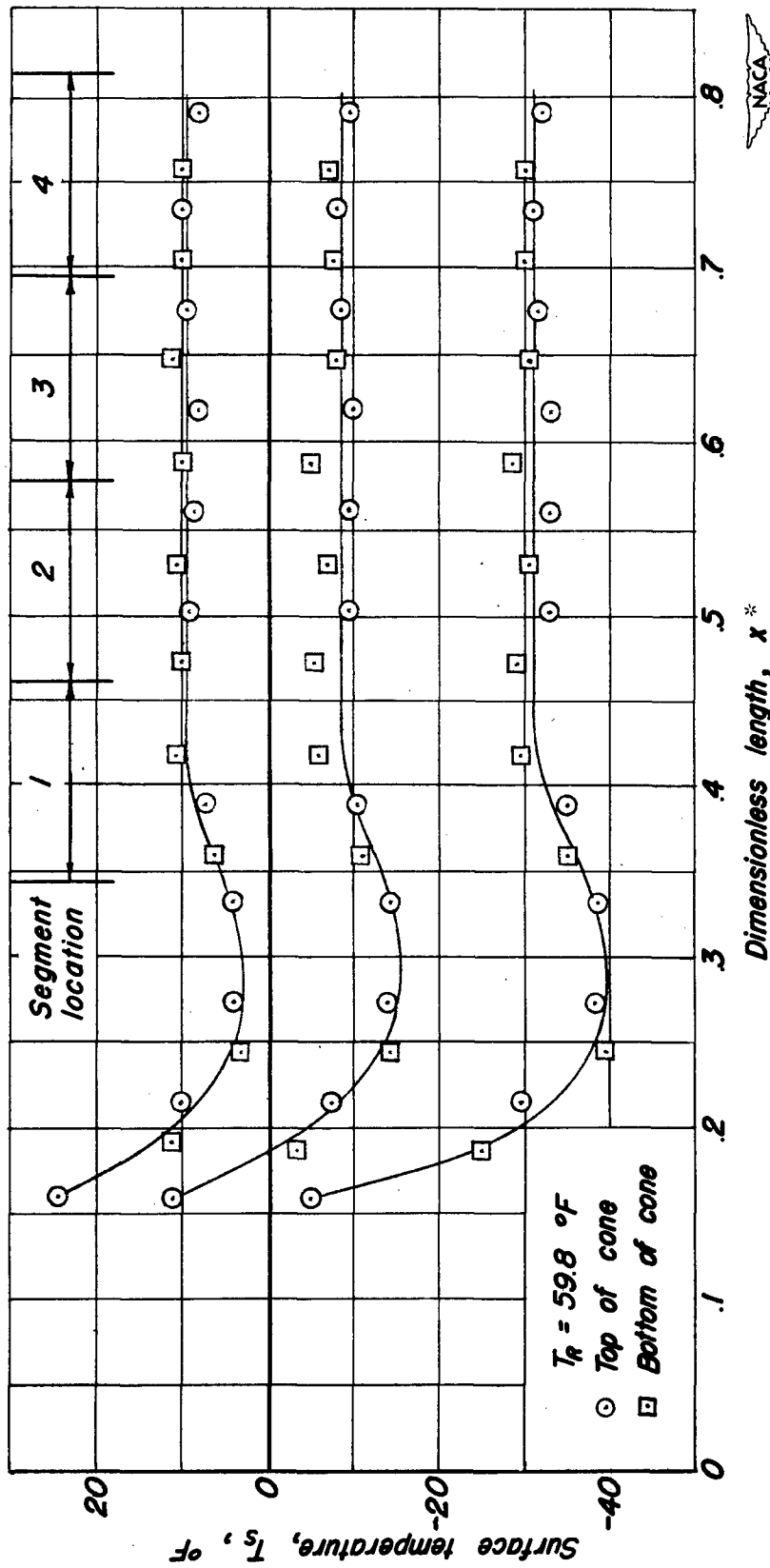
Figure 2. - Diagram of the air cooling system.



(a)  $H_0 = 11.8$  psia.

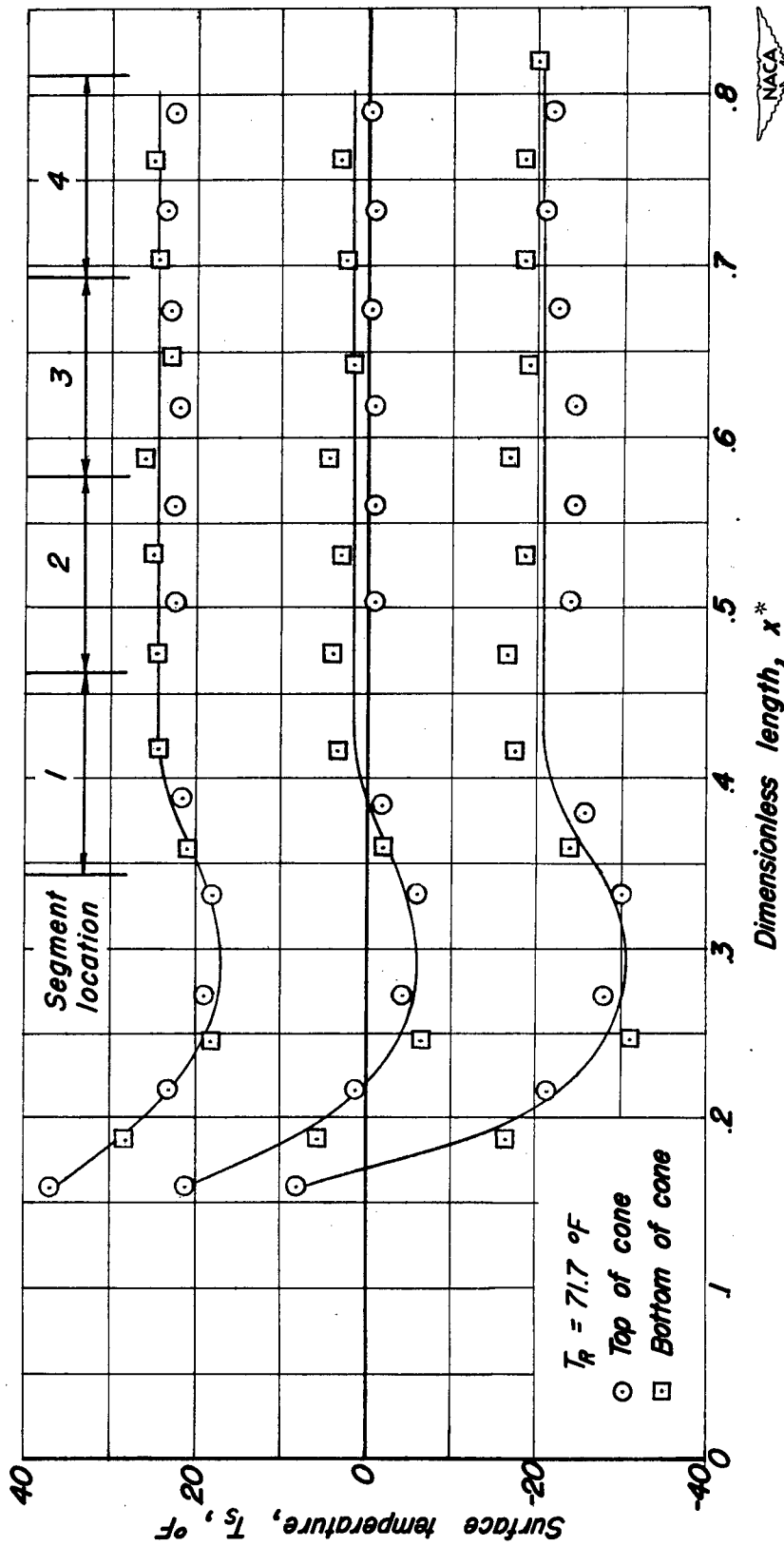
Figure 3. — Surface — temperature distributions for various nominal surface temperatures on a cooled  $20^\circ$  cone with a laminar boundary layer at Mach number 2.02.





(b)  $H_0 = 19.9$  psia.

Figure 3. — Continued.



(c)  $H_0 = 27.9$  psia.

Figure 3. — Concluded.

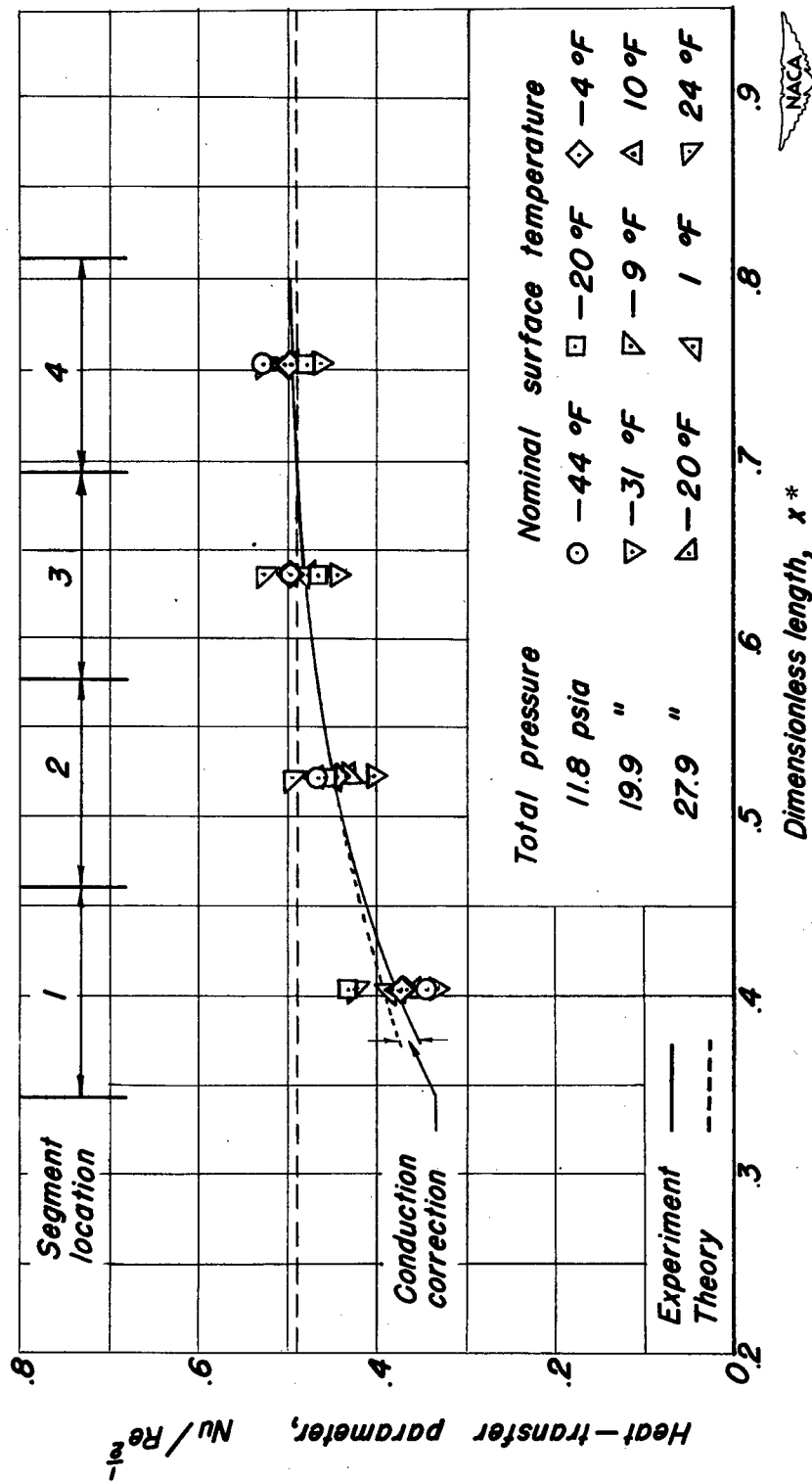


Figure 4.—The variation of local heat-transfer parameter with length for a cooled 20° cone with a laminar boundary layer at Mach number 2.02.

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